Name: ………………………………………………………………………………………

Gosford High School

2023 | Trial HSC examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–31, show relevant mathematical reasoning and/ or calculations

Total Marks

100

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Detach the *Multiple-choice answer sheet* from the last page of this question booklet.

Section II – 90 marks

- Attempt Questions 11-31
- Allow about 2 hours and 45 minutes for this section
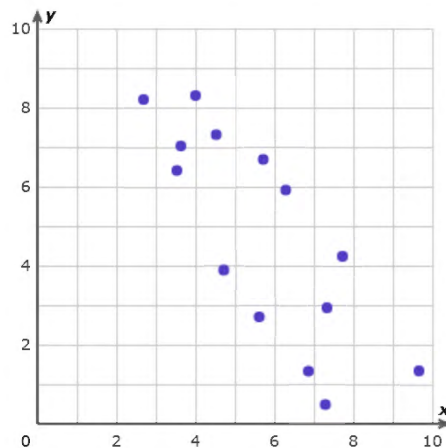
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet *(provided on the last page of the booklet)* for Questions 1 – 10.

_____

1. What is the indefinite integral of $\dfrac{3}{x^2}$?

   A) $-\dfrac{3}{x} + C$

   B) $-\dfrac{3}{2x^3} + C$

   C) $-\dfrac{6}{x^3} + C$

   D) $-\dfrac{6}{x} + C$

2. What is the correlation between the variables in this scatterplot?

   A) Moderate positive correlation
   B) Strong negative correlation
   C) Strong positive correlation
   D) Moderate negative correlation

3. If $f(x) = x^2$ and $g(x) = -1 - (x + 3)^2$ what transformations map the graph of $f(x)$ to $g(x)$?

   A) Horizontally translated 3 units to the left, vertically translated 1 unit up and reflected across the x-axis
   B) Horizontally translated 3 units to the right, vertically translated 1 unit up and reflected across the x-axis
   C) Horizontally translated 3 units to the left, vertically translated 1 unit up and reflected across the y-axis
   D) Horizontally translated 3 units to the right, vertically translated 1 unit down and reflected across the x-axis

4. The Pareto chart shows the orders received by business over 5 months.
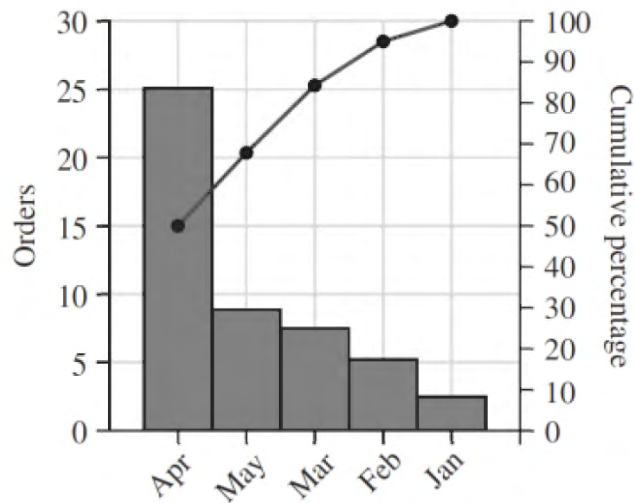
What percentage of orders were received in May?
A) 69%
B) 45%
C) 30%
D) 18%

5. What are the solutions to $|x - 2| = 16$?
A) $x = 14, 18$
B) $x = -14, 18$
C) $x = -14, 14$
D) $x = -18, 18$

6. The graph shows the quadratic function $f(x) = ax^2 + bx + c$.
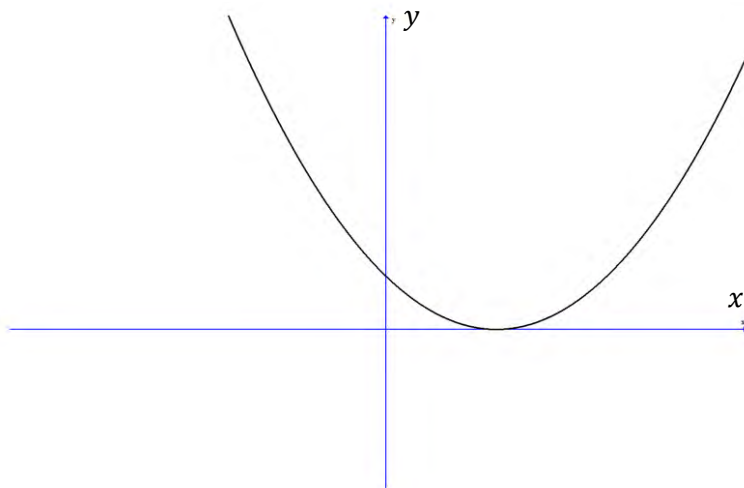
Which of the following are true?
A) $a > 0, c > 0$ and $b^2 - 4ac = 0$
B) $a > 0, c = 0$ and $b^2 - 4ac = 0$
C) $a < 0, c > 0$ and $b^2 - 4ac < 0$
D) $a > 0, c = 0$ and $b^2 - 4ac > 0$

7. A group of 340 students was surveyed. The students were asked whether or not they complete homework daily. The results are recorded in the table below.

	Completes Homework Daily	Does not Complete Homework Daily	Total
Males	93	59	152
Females	150	38	188
Total	243	97	340

A student is selected at random from the group.

What is the approximate probability that the person selected is female OR completes homework daily?

A) 44%
B) 39%
C) 83%
D) 80%

8. The triangle below has an area of $18cm^2$.

x cm $\dfrac{5\pi}{6}$ $2x$ cm   Not to scale
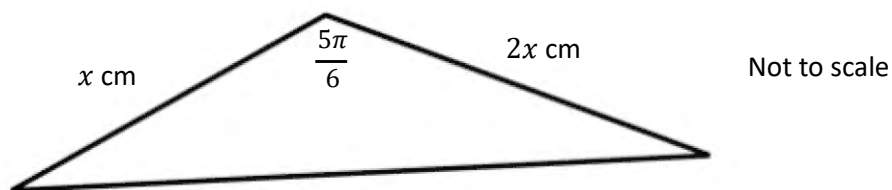
What is the value of x?
A) 3 cm
B) 6 cm
C) 12 cm
D) 24 cm

9. The graph below shows a piecewise function $f(x)$. Find the value of k, shown on the diagram, which satisfies

$$\int_{-4}^{12} f(x)\,dx = 0$$

Not to Scale

(2, 6)

(12, k)

(2, -2)

(-4, -11)
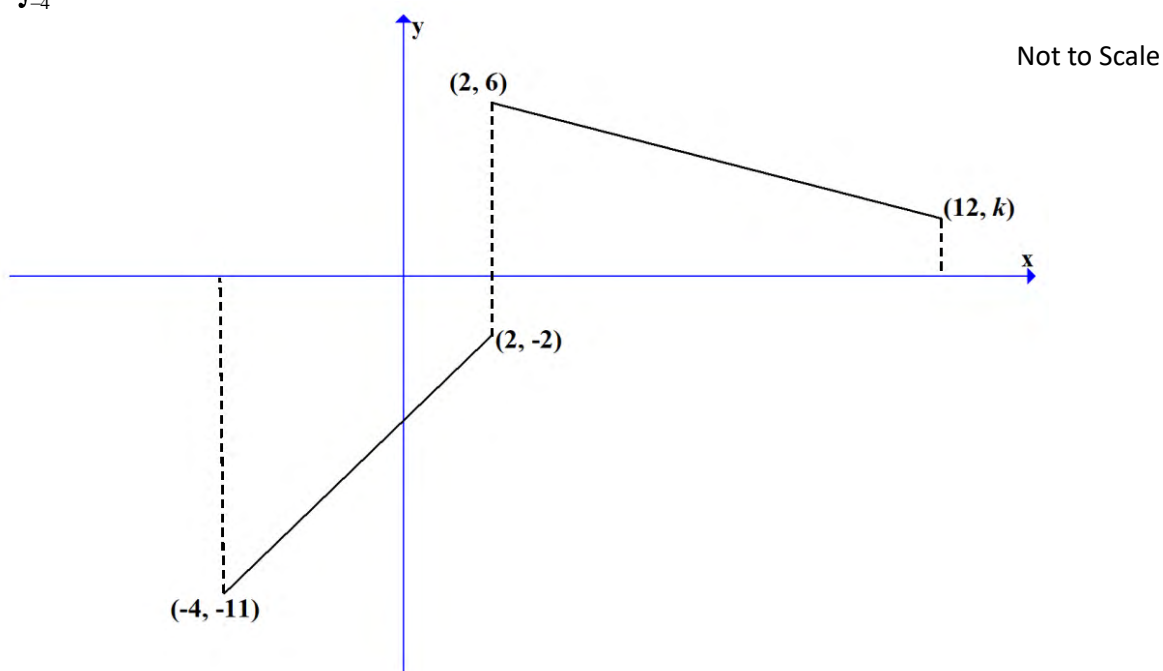
A) 1.8
B) 2
C) 1.55
D) 1

10. How many solutions does $(\cos x)(\tan 2x - 3) = 0$ have between 0 and 2π?

A) 0
B) 2
C) 4
D) 6

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks

Attempt Questions 11–31

Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–23 (51 marks)

Booklet 2 — Attempt Questions 24–31 (39 marks)

Instructions

- **Write your Student Number at the top of this page.**
- **Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided on pages 19-20. If you use this space, clearly indicate which question you are answering**

Question 11   (2 marks)

Express $(3\sqrt{2}+4\sqrt{3})^2$ in the form $a+m\sqrt{6}$ and find the values of a and m.

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Question 12   (2 marks)

The first 3 terms of an arithmetic sequence are $3x^2-15, 3x^2-9, 3x^2-3$. If $T_{18}=\dfrac{265}{3}$, find the value of x.

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Question 13   (6 marks)

Differentiate with respect to x:

a) $\ln(x^2+3x+5)$ **1**

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b) $(x+\sin 2x)^3$ **2**

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c) $\dfrac{1-10x}{(2x-1)^5}$ **3**

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Question 14   (5 marks)

a) Show that the points $A(17,1)$, $B(2,-5)$ and $C(-4,10)$ are the vertices of a right angled triangle.

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b) Hence, find the area of the triangle.

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Questions 15   (2 marks)

a) The probability that Arabella will score more than 100 points in a game of bowling is $\dfrac{21}{30}$. Isabella

states that the probability that Arabella will score less than 100 points in a game of bowling is $\dfrac{9}{30}$.

Is Isabella correct? Give a reason for your answer.

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b) Arabella plays two games of bowling. What is the probability that she scores more than 100
points in the first game and then again in the second game?

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Question 16   (2 marks)

If $\log_e 4 = m$ and $\log_e 5 = p$, write an expression for $\log_e 10$ in terms of m and p.

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Question 17   (5 marks)

A particle is moving in the x-axis. At time t its position is given by $x = t + 1 + \dfrac{1}{1+t}$ where t is measured in seconds.

a) What is the initial position of the particle?   **1**

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b) What is the velocity of the particle at 2 seconds?   **2**

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c) An equation for the acceleration in terms of t.   **2**

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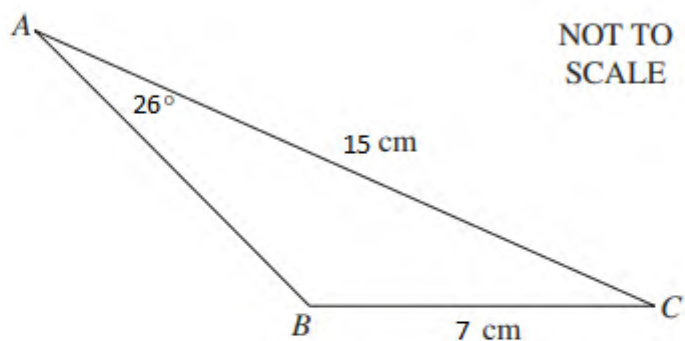
Question 18　(3 marks)

Find the size of the obtuse angle $\angle ABC$, correct to the nearest degree.

NOT TO SCALE

A

$26°$

15 cm

B　7 cm　C

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Question 19　(3 marks)

Find the equation of the normal to the graph $y = 1 + \sin^4 x$ at $x = \dfrac{\pi}{3}$.

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Question 20   (7 marks)

Consider the function $f(x) = \dfrac{1}{30}\left(x^4 - 14x^3 + 40x^2\right)$

a) Find $f'(x)$ **1**

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b) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature (correct to 2 decimal places).   **4**

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Question 20 continued

c) In the space below, sketch the graph of $y = f(x)$ showing all above information and any intercepts. Do not find the points of inflection. **2**

Question 21   (3 marks)

OXY is a sector of a circle with $\angle XOY = \dfrac{5\pi}{18}$ and $OX = OY = 1200$ millimetres.

NOT TO SCALE

How much longer, to the nearest millimetre, is the arc *XY* than the line segment *XY*?

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Bag A contains 1 blue ball and 5 brown balls. Bag B contains 1 blue ball and 3 brown balls. Bag C contains 1 blue ball and 2 brown balls. A ball is chosen at random from Bag A.

If the ball is blue, no further balls are chosen, but if the ball from bag A is brown, then a ball is chosen at random from bag B. If this ball is blue, no further balls are chosen, but if the ball from bag B is brown, then a ball is chosen at random from bag C.

Let the discrete random variable X denote the number of brown balls chosen.

a)   Complete the probability tree. **1**

b)   Complete the probability distribution table: **2**

x	0	1	2	3
$p(x)$				

c)   What is the expected value for the number of brown balls drawn? **2**

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d)   Find in the simplest fraction form the probability that at least one ball of each colour is chosen? **2**

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Question 23   (4 marks)

A random variable is normally distributed with a mean 0 and standard deviation 1. The probability values given in the table for different values of z are represented by the shaded area in the diagram below the table.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

a) The arm span (in metres) for a group of 10 000 residents of a town are normally distributed with   **1**
a mean of 1.68 metres and a standard deviation of 0.24 metres.

Alexa has an arm span of 1.4832 metres.

Calculate the z-score of Alexa's arm span.

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b) Determine how many of the residents will have an arm span less than 1.4832 metres. **3**

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Section II Booklet 1 - extra writing space

If you use this space, clearly indicate which question you are answering.

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Student number

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 24–31 (39 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 32-34. If you use this space, clearly indicate which question you are answering

Question 24   (4 marks)

Spencer, a football super coach, is studying the relationship between the weight in kilograms of players and their vertical jump in metres. Spencer believes that players who weigh less are able to jump higher.

The data from 10 players is shown in the table below.

weight, (w)	84.5	83.3	58.7	74.1	70.7	72.3	75.8	71.8	63.2	65.9
vertical jump (v)	21.9	20.5	20.8	19.9	20.6	18.8	22.8	20.6	20.0	21.8

a) Find the equation of the least-squares regression line in terms of w and v. Give each value correct to two decimal places.

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b) By calculating Pearson's correlation coefficient for the data, correct to three decimal places, justify whether the data confirms the association between weight and vertical jump.   **2**

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Question 25 (6 marks)

A children's picture book is being designed so that each page contains $320\ cm^2$ of print and pictures (shaded area) surrounded completely by a white border as illustrated in the figure below.
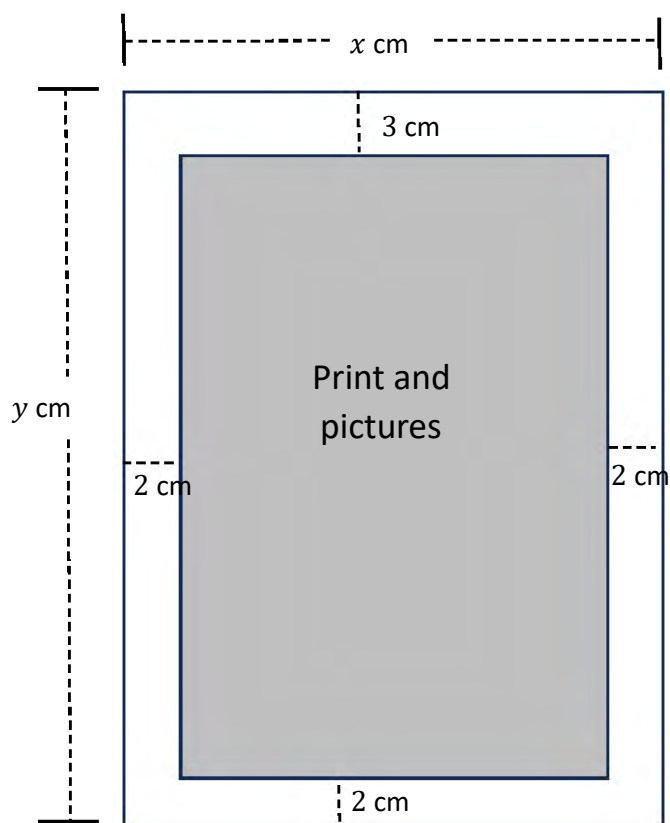
On each page the white border is $2\ cm$ wide on the bottom, left and right of the print area and has a width of $3\ cm$ at the top.

Let x be the width of a page and y be the height of a page in centimetres

a) Show that the area, A, in square centimetres of the whole page is given by $A = x \left[5 + \dfrac{320}{x-4} \right]$ **2**

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b) Determine the width and height of the page which fulfills all the printing requirements and has a minimum area. **4**

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Question 26   (8 marks)

The continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(x-1)+0.3 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a)   Show that $k = \dfrac{3}{5}$ **2**

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b)   Find the mode of the distribution.   **2**

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c) Find the cumulative distribution function $F(x)$ of X **2**

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d) Hence or otherwise find $P(0 < X < 1)$ **2**

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The shaded region ABCD is bounded by the lines $x = -1$, $x = 5$, the curve $y = (x + 1)^2$, the line $y = 6 - 2x$ and the x-axis as shown in the diagram below.
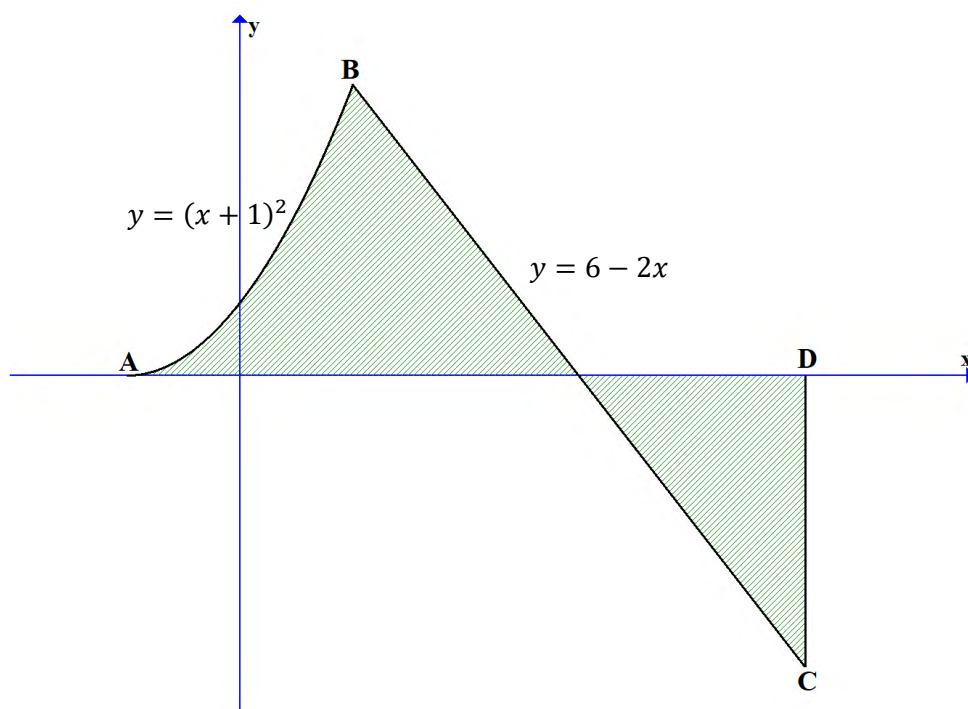
a) Show that B has co-ordinates $(1, 4)$.   **2**

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b) What is the area of the shaded region $ABCD$?   **3**

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Question 28   (4 marks)
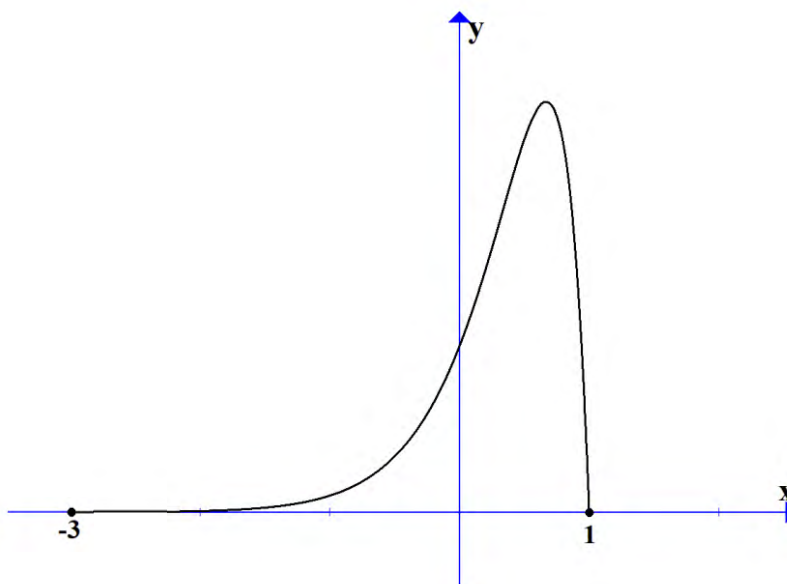
Consider the function defined by $f(x) = e^{3x}(2 - 2x)$ where $-3 \le x \le 1$.

a) Complete the table of values below. Give values correct to 3 decimal places.   **1**

x	-3	-2	-1	0	1
$f(x)$	0.001	0.015			

b) Using the trapezoidal rule with five function values, approximate the area under the curve   **2**
    $y = f(x)$ for $-3 \le x \le 1$

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c) From the diagram, decide whether this approximation is an over-estimate or an under-estimate   **1**
    of the true value of the area under the curve. Justify your answer.

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Question 29 (6 marks)

A railway line is being built between Albury and Ballarat. As of the 1st of January 2023, 80km has been built. Each month 15km of new track is built.

a) Find the total length of the track 1 month, 2 months and 3 months after the 1st of January 2023. Hence, show that this is an arithmetic sequence. **2**

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b) Find an expression for the total length of the track n months after the 1st of January 2023 **1**

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c) The distance from Albury to Ballarat is 392km. In what month and year will the railway line be finished? **2**

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Question 30 (3 marks)

The graphs of $f(x) = -x^2 + 5x$ and $g(x) = x^2 - 9x + 12$ are shown graphed below. Find the area of the shaded region between the 2 curves.

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Question 31　　(4 marks)

The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{x^2}{45} & 0 \le x \le 3 \\[2mm] \dfrac{1}{5} & 3 < x < 4 \\[2mm] \dfrac{1}{3} - \dfrac{x}{30} & 4 \le x \le 10 \\[2mm] 0 & \text{otherwise} \end{cases}$$
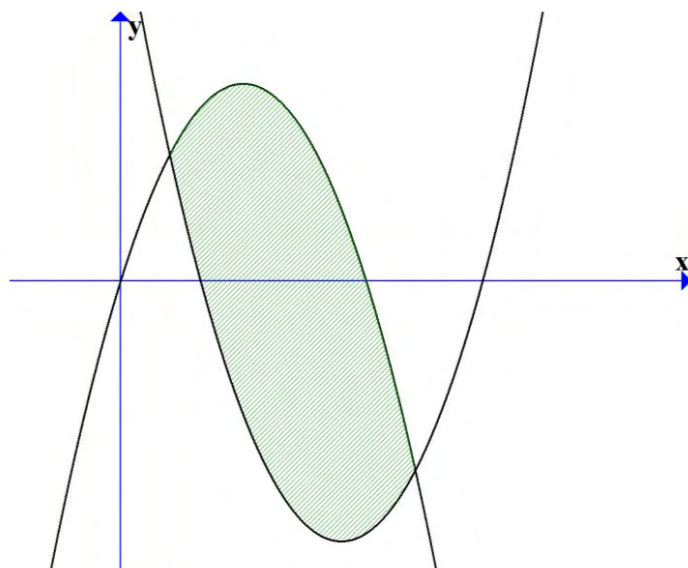
a)　Sketch $f(x)$ for $0 \le x \le 10$　　　　　　　　　　　　　　　　　**2**
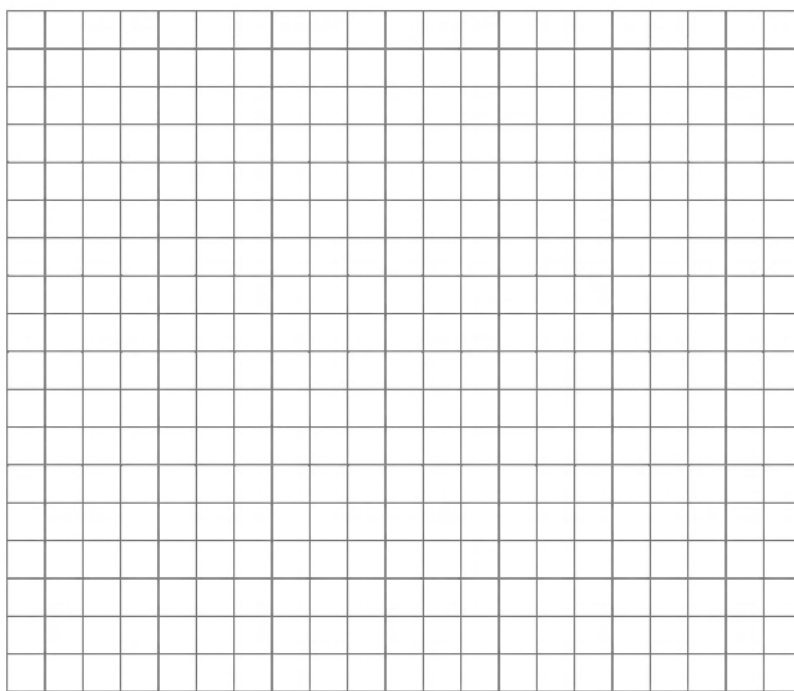
b)　Find $P(X \le 8)$　　　　　　　　　　　　　　　　　　　　　　**2**

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Section II Booklet 2 - extra writing space

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Gosford High School

2023 Trial HSC examination

Mathematics Advanced

Solutions

2023 Year 12 HSC Advanced Examination

Section 1: Solutions

1. A ● B ○ C ○ D ○
2. A ○ B ○ C ○ D ●
3. A ● B ○ C ○ D ○
4. A ○ B ○ C ○ D ●
5. A ○ B ● C ○ D ○
6. A ● B ○ C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ● C ○ D ○
9. A ● B ○ C ○ D ○
10. A ○ B ○ C ○ D ●

1. What is the indefinite integral of $\dfrac{3}{x^2}$?

A) $-\dfrac{3}{x}+C$

$$= 3\int x^{-2}dx$$

$$= 3\dfrac{x^{-1}}{-1}+C$$

$$= -\dfrac{3}{x}+C$$

2. What is the correlation between the variables in this scatterplot?
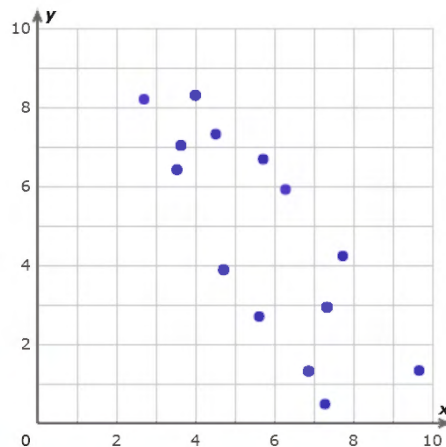
D) Moderate negative correlation

3. If $f(x) = x^2$ and $g(x) = 1 - (x+3)^2$ what transformations map the graph of $f(x)$ to $g(x)$?

A) Horizontally translated 3 units to the left, vertically translated 1 unit up and reflected across the x-axis

4. The Pareto chart shows the orders received by business over 5 months.
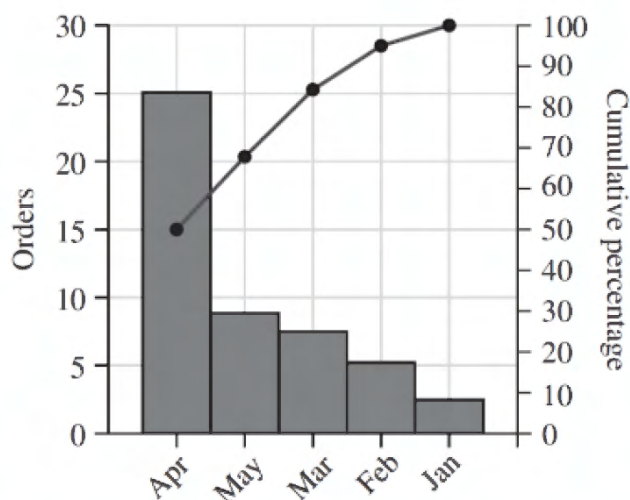
What percentage of orders were received in May?

D

$85\% - 67\% = 18\%$

5. What are the solutions to $|x - 2| = 16$?

B) $x = -14, 18$

$$x - 2 = \pm 16$$
$$x = -16 + 2, 16 + 2$$
$$x = -14, 18$$

6. The graph shows the quadratic function $f(x) = ax^2 + bx + c$.

Which of the following are true?

A) $a > 0, c > 0$ and $b^2 - 4ac = 0$

7. A group of 340 students was surveyed. The students were asked whether or not they complete homework daily. The results are recorded in the table below.

	Completes Homework Daily	Does not Complete Homework Daily	Total
Males	93	59	152
Females	150	38	188
Total	243	97	340

A student is selected at random from the group.

What is the approximate probability that the person selected is female OR completes homework daily?

C) 83%

$$\frac{243 + 38}{340} = 82.65\%$$
$$\approx 83\%$$

8. The triangle below has an area of $18cm^2$.

5π —
6

x cm $2x$ cm   Not to scale
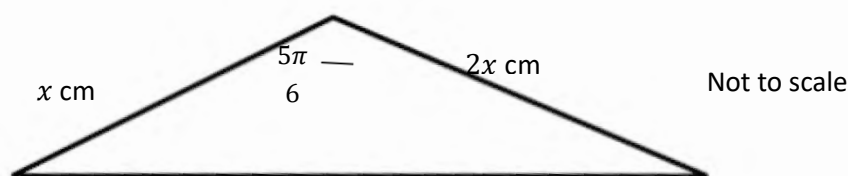
What is the value of x?

B) 6 cm

$$A = \frac{1}{2}ab\sin C$$

$$18 = \frac{1}{2} \times 2x \times x \times \sin\frac{5\pi}{6}$$

$$x^2 = \frac{18}{\sin\frac{5\pi}{6}}$$

$$x = 6$$

9. The graph below shows a piecewise function $f(x)$. Find the value of k, shown on the diagram, which satisfies

$$\int_{-4}^{12} f(x)dx = 0$$

(2, 6)   Not to Scale

(12, k)

(2, -2)

(-4, -11)
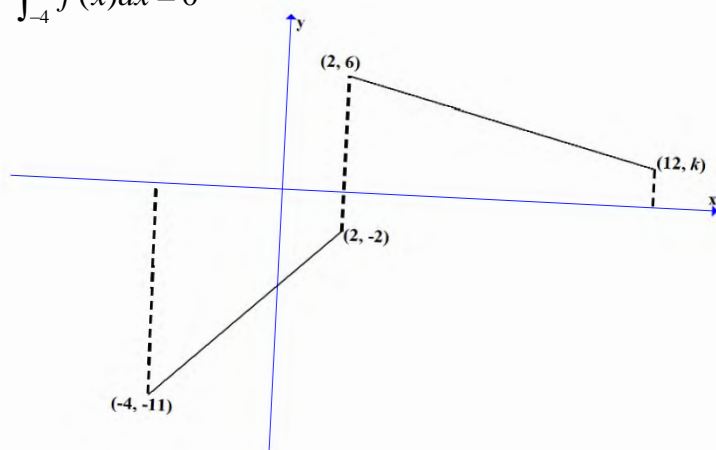
A) 1.8

Equating trapeziums:

$$\frac{6}{2} \times (11+2) = \frac{10}{2} \times (6+k)$$

$$39 = 5(6+k)$$

$$6+k = 7.8$$

$$k = 1.8$$

10. How many solutions does $(\cos x)(\tan 2x - 3) = 0$ have between 0 and 2π?

D) 6

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\tan 2x - 3 = 0$$

$$\tan 2x = 3$$

$$2x = 1.249, 4.391, 7.532, 10.674$$

$$x = 0.625, 2.196, 3.766, 5.337$$

Mathematics Advanced

Section II Answer Booklet 1

Solutions

Question 11   (2 marks)

Express $(3\sqrt{2}+4\sqrt{3})^2$ in the form $a+m\sqrt{6}$ and find the values of a and m.

Solution:

$9\times 2+12\sqrt{6}+12\sqrt{6}+16\times 3$

$=18+48+24\sqrt{6}$

$=66+24\sqrt{6}$

$\therefore a=66, m=24$

Marks	Guideline
2	Correct response including stating values of a and m
1	For correct expansion
Marker's Comment	Students commonly made mistakes with expanding, either multiplying by the square of the number under the root or missing the middle term.

Question 12   (2 marks)

The first 3 terms of an arithmetic sequence are $3x^2-15, 3x^2-9, 3x^2-3$. If $T_{18}=\dfrac{265}{3}$, find the value of x.

$d=3x^2-9-(3x^2-15)$

$d=6$

$T_n=a+(n-1)d$

$\dfrac{265}{3}=3x^2-15+(18-1)\times 6$

$\dfrac{265}{3}=3x^2+87$

$3x^2=\dfrac{4}{3}$

$x^2=\dfrac{4}{9}$

$x=\pm\dfrac{2}{3}$

Marks	Guideline
2	Correct responses
1	For progress towards solution
Marker's Comment	Many students struggled finding the common difference. When common difference was found, algebraic errors when solving for x occurred often. When square rooting, keep both possible values (positive and negative)

Question 13   (6 marks)

Differentiate with respect to x:

a) $\ln(x^2+3x+5)$

$\dfrac{dy}{dx}=\dfrac{2x+3}{x^2+3x+5}$

Marks	Guideline
1	Correct answer
Marker's Comment	Most students did this well.

b) $(x+\sin 2x)^3$

$\dfrac{dy}{dx}=3(1+2\cos 2x)(x+\sin 2x)^2$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Common issues included not correctly implementing the chain rule or not correctly finding f'(x)

c) $\dfrac{1-10x}{(2x-1)^5}$

$u = 1-10x$
$u' = -10$

$v = (2x-1)^5$
$v' = 5 \times 2 \times (2x-1)^4$
$\quad = 10(2x-1)^4$

$\dfrac{dy}{dx} = \dfrac{-10(2x-1)^5 - (1-10x)(10)(2x-1)^4}{(2x-1)^{10}}$

$\quad = \dfrac{-20x+10-10+100x}{(2x-1)^6}$

$\quad = \dfrac{80x}{(2x-1)^6}$

Marks	Guideline
3	Correct response
2	Equation not fully simplified
1	Attempts use of the quotient or product rule
Marker's Comment	Most students recognised the need for the quotient rule. Not many were able to fully simplify the derivative without errors.

Question 14 (5 marks)

a) Show that the points $A(17, 1)$, $B(2, -5)$ and $C(-4, 10)$ are the vertices of a right angled triangle.

$m_{AB} = \dfrac{1--5}{17-2}$

$\quad = \dfrac{6}{15}$

$m_{BC} = \dfrac{10--5}{-4-2}$

$\quad = -\dfrac{15}{6}$

$\therefore m_{AB} \perp m_{BC}$

$\therefore \Delta ABC$ is right angled

Marks	Guideline
3	Correct response
2	Both correct gradients found but no conclusions stated OEM
1	One correct gradient found or one correct side length found in exact form
Marker's Comment	Many students seemed confused about how to approach this question or could not accurately remember/apply the gradient or distance formulas.

b) Hence, find the area of the triangle.

$A = \dfrac{1}{2}bh$

$\quad = \dfrac{1}{2} \times \sqrt{(17-2)^2 + (1--5)^2} \times \sqrt{(2+4)^2 + (-5-10)^2}$

$\quad = 130.5 \, units^2$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Students who got part a correct, generally did very well at this question.

Questions 15   (2 marks)

a) The probability that Arabella will score more than 100 points in a game of bowling is $\dfrac{21}{30}$. Isabella

states that the probability that Arabella will score less than 100 points in a game of bowling is $\dfrac{9}{30}$.

Is Isabella correct? Give a reason for your answer.   **1**

No. P(100) must be included in the probabilities somewhere.

Marks	Guideline
1	Correct answer
Marker's Comment	Many students tripped up by this question, thinking the 2 statements were complementary events when they were not.

b) Arabella plays two games of bowling. What is the probability that she scores more than 100   **1**
points in the first game and then again in the second game?

$$P(100,100) = \dfrac{21}{30} \times \dfrac{21}{30}$$
$$= \dfrac{49}{100}$$

Marks	Guideline
1	Correct answer
Marker's Comment	Done very well.

Question 16   (2 marks)

If $\log_e 4 = m$ and $\log_e 5 = p$, write an expression for $\log_e 10$ in terms of m and p.

$\log_e 2^2 = m$

$2\log_e 2 = m$

$\log_e 2 = \dfrac{m}{2}$

$\log_e 10 = \log_e 5 + \log_e 2$

$\qquad = p + \dfrac{m}{2}$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Generally well done. Some students messed up the logarithmic laws, e.g., $\log_e 20 = \log_e 4 \times \log_e 5 \therefore \log_e 10 = \dfrac{mp}{2}$

Question 17   (5 marks)

A particle is moving in the x-axis. At time t its position is given by $x = t + 1 + \dfrac{1}{1+t}$ where t is measured in

seconds.

a) What is the initial position of the particle?   **1**

When $t = 0 \rightarrow x = 0 + 1 + \dfrac{1}{1+0}$

$\qquad\qquad x = 2$ units

Marks	Guideline
1	Correct answer
Marker's Comment	Well done

b) What is the velocity of the particle at 2 seconds? **2**

$$\frac{dy}{dx} = v = 1 + -1(1+t)^{-2}$$

$$= 1 - \frac{1}{(1+t)^2}$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9} \, units/s$$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Generally well done. Some students used the quotient rule on $\frac{1}{1+t}$ and ended up with horrible mess.

c) An equation for the acceleration in terms of t. **2**

$$\frac{dv}{dt} = a = 2(1+t)^{-3}$$

$$= \frac{2}{(1+t)^3}$$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Generally well done but similar problems to part (b)

Question 18   (3 marks)

Find the size of the obtuse angle $\angle ABC$, correct to the nearest degree.

NOT TO SCALE
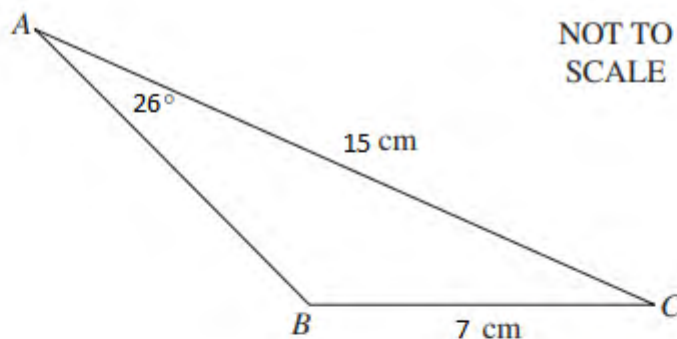
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{15} = \frac{\sin 26}{7}$$

$$\sin B = \frac{15 \times \sin 26}{7}$$

$$B = 180 - \sin^{-1}\left(\frac{15 \times \sin 26}{7}\right)$$

$$= 110°$$

Marks	Guideline
3	Correct response
2	Correctly used Sine Rule but did not find obtuse angle
1	Attempt at using the Sine Rule OEM
Marker's Comment	Generally well done although MANY students failed to find the obtuse angle. Some students found $\frac{15\sin 26°}{7} = 0.93\ldots$ $\therefore B = 0°56' \Rightarrow$ obtuse angle $= 179°3'$

Question 19   (3 marks)

Find the equation of the normal to the graph $y = 1 + \sin^4 x$ at $x = \dfrac{\pi}{3}$.

$$\frac{dy}{dx} = 4(\cos x)(\sin^3 x)$$

$$m_T = 4\cos\frac{\pi}{3}\sin^3\frac{\pi}{3}$$

$$= 4 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{4}$$

$$m_N = -\frac{4}{3\sqrt{3}}$$

$$x = \frac{\pi}{3},\ y = \frac{25}{16}$$

$$y - \frac{25}{16} = \frac{-4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)$$

$$y = \frac{-4\sqrt{3}}{9}x + \frac{4\pi\sqrt{3}}{27} + \frac{25}{16}$$

$$192\sqrt{3}x + 432y - 675 - 64\sqrt{3}\pi = 0$$

$$y = -0.7698x + 2.3686$$

Marks	Guideline
3	Correct response
2	y=value found and substituted in OEM
1	Derivative found successfully
Marker's Comment	Generally well done. Some couldn't find the derivative properly e.g. $\dfrac{dy}{dx} = 4\cos^3 x$

Question 20   (7 marks)

Consider the function $f(x) = \dfrac{1}{30}\left(x^4 - 14x^3 + 40x^2\right)$

a) Find $f'(x)$

$$f'(x) = \frac{1}{30}(4x^3 - 42x^2 + 80x)$$

Marks	Guideline
1	Correct answer
Marker's Comment	Generally well done.

b) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature (correct to 2 decimal places). **4**

$$f'(x) = 0 = \frac{1}{30}(4x^3 - 42x^2 + 80x)$$

$$4x^3 - 42x^2 + 80x = 0$$

$$2x(2x^2 - 21x + 40) = 0$$

$$x = 0, x = \frac{21 \pm \sqrt{21^2 - 4 \times 2 \times 40}}{2 \times 2}$$

$$x = 0, 8, 2.5$$

$$f(0) = 0, f(8) = \frac{-256}{15} \approx -17.067, f(2.5) = \frac{75}{32} \approx 2.344$$

$$f''(x) = \frac{1}{30}(12x^2 - 84x + 80)$$

$$f''(0) = \frac{8}{3}$$

$\therefore$ minimum at $(0,0)$

$$f''(8) = \frac{88}{15}$$

$\therefore$ minimum at $\left(8, \frac{-256}{15}\right)$

$$f''(2.5) = \frac{-11}{6}$$

$\therefore$ maximum at $\left(2.5, \frac{75}{32}\right)$

Marks	Guideline
4	Correct response
3	One arithmetic error or omits y ordinate
2	Finds the x-ordinates but does not justify the nature
1	Work towards solution
Marker's Comment	Many students failed to find $x = 0$ as a stationary point.

Question 20 continued

c) In the space below, sketch the graph of $y = f(x)$ showing all above information and any intercepts. Do not find the points of inflection. **2**
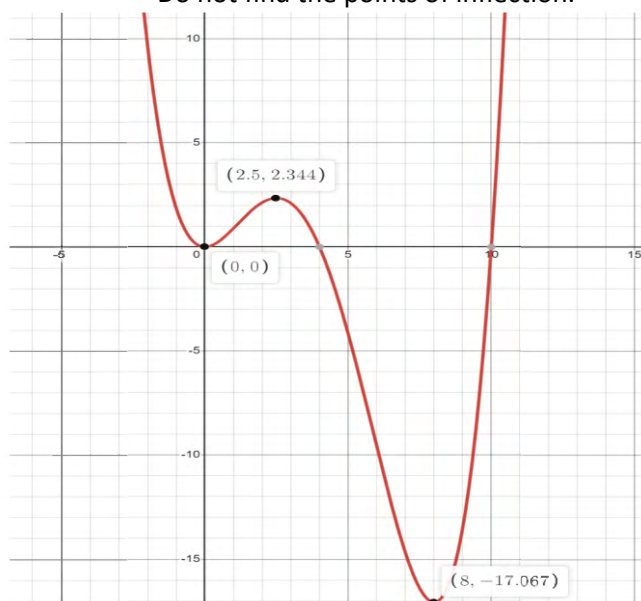
Marks	Guideline
2	Correct graph
1	For progress towards solution (missing intercept or stationary point intercepts)
Marker's Comment	Mostly well done, although some student's answers didn't match their part (b) answers.

Question 21   (3 marks)

OXY is a sector of a circle with $\angle XOY = \dfrac{5\pi}{18}$ and $OX = OY = 1200$ millimetres.

NOT TO
SCALE
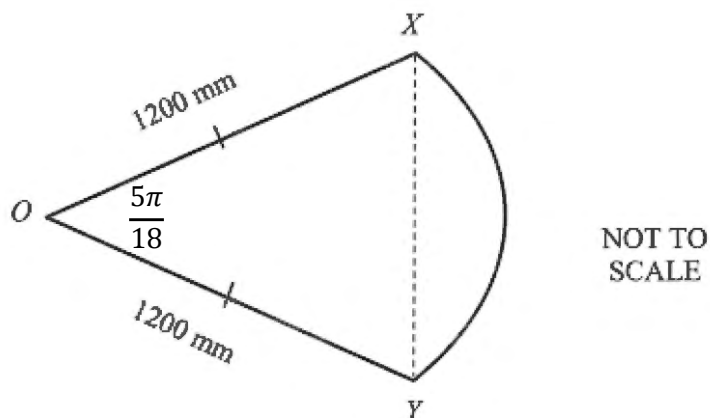
How much longer, to the nearest millimetre, is the arc XY than the line segment XY?

$l = r\theta$

$= 1200 \times \dfrac{5\pi}{18}$

$= 1047.198 mm$

$XY^2 = 1200^2 + 1200^2 - 2 \times 1200^2 \times \cos\dfrac{5\pi}{18}$

$XY^2 = 1028771.684$

$XY = 1014.284 mm$

$l - XY = 1047.198 - 1014.284$

$= 32.914$

$= 33 mm$ (to the nearest mm)

Marks	Guideline
3	Correct response
2	Correct application of the arc length and cosine rule formula
1	Found arc length or length of XY correctly.
Marker's Comment	Mostly well done. Some students used degrees for the arc length or used the cosine rule incorrectly.

Question 22 (7 marks)

Bag A contains 1 blue ball and 5 brown balls. Bag B contains 1 blue ball and 3 brown balls. Bag C contains 1 blue ball and 2 brown balls. A ball is chosen at random from Bag A.

If the ball is blue, no further balls are chosen, but if the ball from bag A is brown, then a ball is chosen at random from bag B. If this ball is blue, no further balls are chosen, but if the ball from bag B is brown, then a ball is chosen at random from bag C.

Let the discrete random variable X denote the number of brown balls chosen.

a) Complete the probability tree.   1
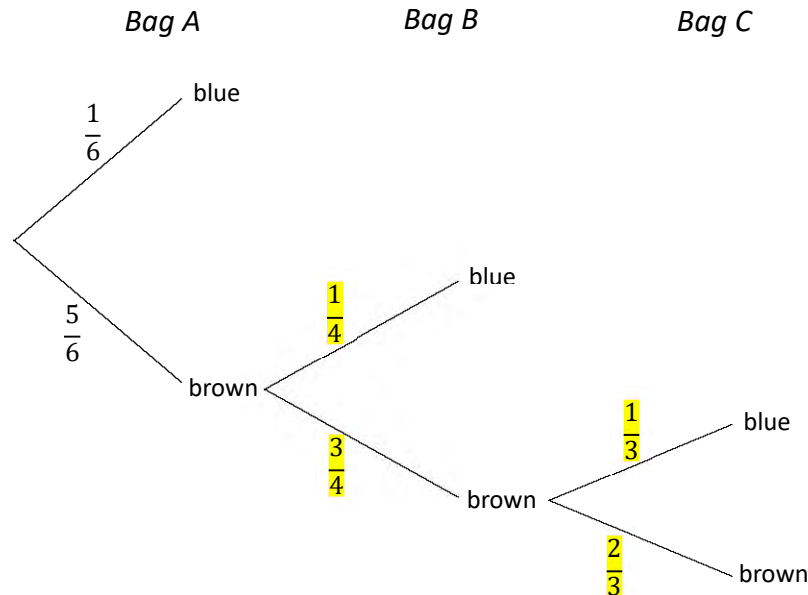
b) Complete the probability distribution table: 2

x	0	1	2	3
$p(x)$	$\dfrac{1}{6}$	$\dfrac{5}{24}$	$\dfrac{5}{24}$	$\dfrac{5}{12}$

c) What is the expected value for the number of brown balls drawn?   2

$$E(X) = 0 \times \frac{1}{6} + 1 \times \frac{5}{24} + 2 \times \frac{5}{24} + 3 \times \frac{5}{12}$$
$$= 1.875$$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Generally well done.

d) Find in the simplest fraction form the probability that at least one ball of each colour is chosen?   2

$$\frac{5}{24} + \frac{5}{24} = \frac{10}{24} = \frac{5}{12}$$

Marks	Guideline
2	Correct response
1	For progress towards solution
Marker's Comment	Generally well done. Some students didn't use their (correct) answers from the table, but calculated the probabilities (incorrectly) again.

Question 23 (4 marks)

A random variable is normally distributed with a mean 0 and standard deviation 1. The probability values given in the table for different values of z are represented by the shaded area in the diagram below the table.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

a) The arm span (in metres) for a group of 10 000 residents of a town are normally distributed with a mean of 1.68 metres and a standard deviation of 0.24 metres.   **1**

Alexa has an arm span of 1.4832 metres.

Calculate the z-score of Alexa's arm span.

$$z = \frac{x - \mu}{\sigma}$$
$$= \frac{1.4832 - 1.68}{0.24}$$
$$= -0.82$$

Marks	Guideline
1	Correct answer
Marker's Comment	Well done.

b) Determine how many of the residents will have an arm span less than 1.4832 metres.   **3**

$$P(Z < -0.82) = P(Z > 0.82)$$
$$= 1 - P(Z < 0.82)$$
$$= 1 - 0.7939$$
$$= 0.2061$$
$$10000 \times 0.2061 = 2061 \text{ residents}$$

Marks	Guideline
3	Correct response
2	Correctly calculates $P(Z < -0.82) = 0.2061$
1	Work towards solution
Marker's Comment	Mostly well done, although some students missed the $-$ sign to get 7939 residents.

Gosford High School Trial HSC

Mathematics Advanced

Section II Answer Booklet 2

SOLUTIONS

Question 24   (4 marks)

Spencer, a football super coach, is studying the relationship between the weight in kilograms of players and their vertical jump in metres. Spencer believes that players who weigh less are able to jump higher.

The data from 10 players is shown in the table below.

weight, (w)	84.5	83.3	58.7	74.1	70.7	72.3	75.8	71.8	63.2	65.9
vertical jump (v)	21.9	20.5	20.8	19.9	20.6	18.8	22.8	20.6	20.0	21.8

Marks	Guideline
2	Correct response with v and w included in equation
1	For correctly finding A and B

a) Find the equation of the least-squares regression line in terms of w and v. Give each value correct to two decimal places.

Marker's Comment	Mostly well done. It was evident that some students did not know how to do this on their calculator. A very common and concerning error was rounding to 2 decimal places. This is a Stage 3 / 4 concept.

$v = A + Bw$

$A = 18.75, B = 0.03$

$v = 0.03w + 18.75$

b) By calculating Pearson's correlation coefficient for the data, correct to three decimal places, justify whether the data confirms the association between weight and vertical jump. **2**

$r = 0.199$

Pearson's correlation coefficient indicates a weak positive correlation at best. This does not support Spencer's statement.

Marks	Guideline
2	Correct response with relevant statement about Spencer's statement
1	For finding r or making a correct statement.
Marker's Comment	Mixed results. Better responses found Pearson's correlation coefficient and correctly interpreted that the weak positive correlation is not strong enough to support Spencer's statement. Some responses incorrectly stated that there was no correlation between the variables.

Question 25 (6 marks)

A children's picture book is being designed so that each page contains 320 cm^2 of print and pictures (shaded area) surrounded completely by a white border as illustrated in the figure below.
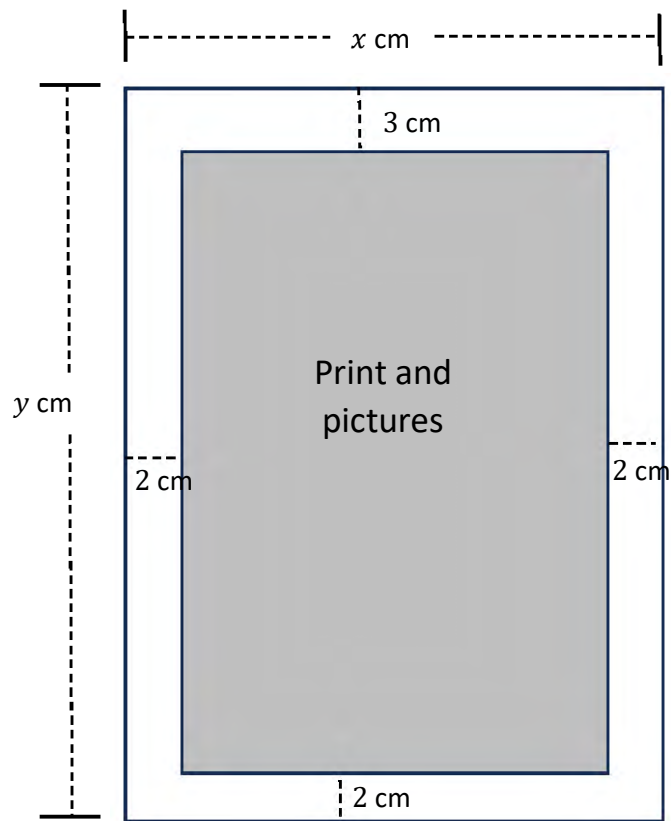
On each page the white border is 2 cm wide on the bottom, left and right of the print area and has a width of 3 cm at the top.

Let x be the width of a page and y be the height of a page in centimetres

a) Show that the area, A, in square centimetres of the whole page is given by $A = x\left[5 + \dfrac{320}{x-4}\right]$ **2**

$$(x-4)(y-5) = 320$$

$$y - 5 = \frac{320}{x-4}$$

$$y = 5 + \frac{320}{x-4}$$

$$A = xy$$

$$= x\left[5 + \frac{320}{x-4}\right]$$

$$\therefore \text{ as required}$$

Marks	Guideline
2	Correct solution showing all necessary steps, including expression for area of shaded space and Area of whole page.
1	For demonstrating some necessary steps.
Marker's Comment	Very poorly done. Much of the cohort lacks the algebra skills to manipulate the area of the shaded rectangle to get the desired formula. A common error was writing x-4 * y-5 = 320 rather than (x-4)(y-5)-320. These are not the same thing.

b) Determine the width and height of the page which fulfills all the printing requirements and has a minimum area.

$$A = x\left[5 + \frac{320}{x-4}\right]$$

$$u = x$$

$$u' = 1$$

$$v = 5 + \frac{320}{x-4} = 5 + 320(x-4)^{-1}$$

$$v' = -320(x-4)^{-2}$$

$$A' = vu' + uv'$$

$$= 5 + \frac{320}{x-4} - \frac{320x}{(x-4)^2} = 0$$

$$5(x-4)^2 + 320(x-4) - 320x = 0$$

$$5(x^2 - 8x + 16) + 320x - 1280 - 320x = 0$$

$$5x^2 - 40x + 80 - 1280 = 0$$

$$x^2 - 8x - 240 = 0$$

$$(x-20)(x+12) = 0$$

$$x = 20, x = -12$$

Ignore negative solution

x	19	20	21
$A'(x)$	$-\dfrac{31}{45}$	0	$\dfrac{165}{289}$

$\therefore$ maximum at x=20

$$y = 5 + \frac{320}{20-4}$$

$$= 25$$

$\therefore$ width and height needed to make maximum area are 20cm and 25cm

Marks	Guideline
4	Correct height and width and proof that it is a minimum point
3	Correct height and width without proving as a minimum
2	Correctly finds x coordinate but does not find y OEM
1	Differentiates A equation correctly OEM
Marker's Comment	Poorly done by most of the cohort. Better responses clearly set out and identified each stage of the problem, differentiated correctly and determined the nature of the found stationary point to determine it gives a minimum area. Some very common errors were: • Incorrectly differentiating.

	• Changing the problem entirely to an easier problem and then failing to answer that problem correctly.
	• Not checking the nature of stationary points.
	• Unclear working out and progression.
	• Using an unlabelled table of values to determine the nature of stationary points, or a table with no values in it.
	• Not writing the units for the dimensions.
	• Having a negative length.

Question 26   (8 marks)

The continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(x-1)+0.3 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a)   Show that $k = \dfrac{3}{5}$ **2**

$$\int_0^2 kx(x-1)+0.3\,dx$$

$$= \int_0^2 kx^2 - kx + 0.3\,dx$$

$$= \left[\frac{kx^3}{3} - \frac{kx^2}{2} + 0.3x \right]_0^2$$

$$= \frac{8k}{3} - \frac{4k}{2} + 0.6 = 1$$

$$8k - 6k + 1.8 = 3$$

$$2k = 1.2$$

$$k = \frac{3}{5}$$

$$\therefore \text{ as required}$$

Marks	Guideline
2	Correct solution showing all necessary steps, including substitution into integration and rearranging for k
1	For demonstrating some necessary steps.
Marker's Comment	Very poor responses across the board. This is a very simple and common 2 mark "show that" question. Better responses recognised that the integral of the PDF over its bounds is equal to 1. These responses then solved for k. Some common errors were not recognising this fact and incorrectly integrating.

b)   Find the mode of the distribution.   **2**

$$f(x) = \frac{3}{5}x^2 - \frac{3}{5}x + 0.3$$

$$f'(x) = \frac{6}{5}x - \frac{3}{5} = 0$$

$$\frac{6}{5}x = \frac{3}{5}$$

$$x = \frac{1}{2}$$

$$f''(x) = \frac{6}{5}$$

$$\therefore \text{ concave up at all values}$$

$$\therefore x = \frac{1}{2} \text{ is a minimum}$$

$$f(0) = 0.3$$

$$f(2) = \frac{3}{5} \times 4 - \frac{3}{5} \times 2 + 0.3$$

$$= 1.5$$

$$\therefore \text{ mode at x=2}$$

Marks	Guideline
2	Correct solution
1	For progress towards solution
Marker's Comment	Very poorly done. Most of the attempted responses only found the stationary point and did not test its nature to see that it is a minimum, not a maximum. Exemplary responses did this and then tested the end points to find that x=2 is the mode.

Question 26 continued

c) Find the cumulative distribution function $F(x)$ of X **2**

$$F(x) = \int_0^x \frac{3}{5}x^2 - \frac{3}{5} + 0.3 \, dx$$

$$= \frac{3}{5} \times \frac{x^3}{3} - \frac{3}{5} \times \frac{x^2}{2} + 0.3x \text{ for } 0 \leq x \leq 2$$

$$= \frac{x^3}{5} - \frac{3x^2}{10} + \frac{3x}{10}$$

Marks	Guideline
2	Correct solution
1	For progress towards solution
Marker's Comment	Many students need to revise a CDF. CDFs is the integral between the lower limit and a variable (typically *x*). There is no + C and should be expressed in simplest form.

d) Hence or otherwise find $P(0 < X < 1)$ **2**

$$P(0 < X < 1) = P(X < 1)$$

$$= F(1)$$

$$= \frac{1}{5} - \frac{3}{10} + \frac{3}{10}$$

$$= \frac{1}{5}$$

Marks	Guideline
2	Correct solution
1	For progress towards solution
Marker's Comment	Many students did not use the result of part (c) and integrated between 0 and 1. A CDF tells us the cumulative probability for all values up to X = x. While reperforming the integration step is also correct they wasted time by not taking advantage of the previous answer. Many students did not reflect upon their answer and provided probabilities greater than 1. Always check the reasonableness of your answer and in this case a number greater than one should alert you to an error. Many students did not use correct notation. Students should communicate that they are finding $P(X < 1)$.

Question 27   (5 marks)

The shaded region ABCD is bounded by the lines $x = -1$, $x = 5$, the curve $y = (x + 1)^2$, the line $y = 6 - 2x$ and the x-axis as shown in the diagram below.

a)   Show that B has co-ordinates $(1, 4)$.   **2**

$(x+1)^2 = 6-2x$ $y = (x + 1)^2$

$x^2 + 2x + 1 = 6 - 2x$

$x^2 + 4x - 5 = 0$ $y = 6 - 2x$
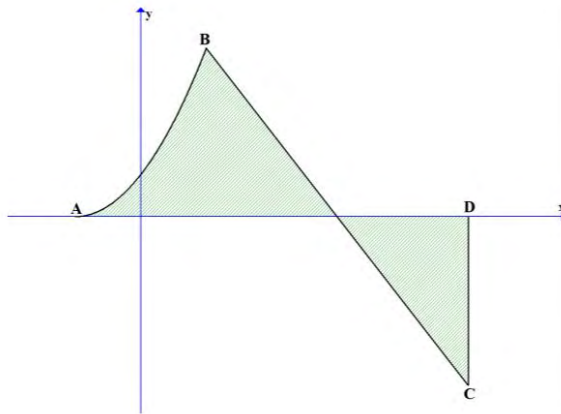
$(x+5)(x-1) = 0$

$x = -5, x = 1$

B must be between -1 and 5, so x=1

$y = (1+1)^2 = 4$

$\therefore$ intersection at $(1,4)$

Marks	Guideline
2	Correct solution
1	For progress towards solution
Marker's Comment	Many students need to revise how to find the point of intersection. Many students will need to revise the methods to solve quadratic equations. "Show that" questions require you to *show* all steps, including the substitution of the found *x* value to find *y*.

b)   What is the area of the shaded region $ABCD$?   **3**

$$\int_{-1}^{1}(x+1)^2\,dx+\frac{1}{2}\times 2\times 4+\frac{1}{2}\times 2\times 4$$

$$=\left[\frac{(x+1)^3}{3}\right]_{-1}^{1}+4+4$$

$$=\frac{8}{3}+4+4$$

$$=\frac{32}{3}\,units^2$$

Marks	Guideline
3	Correct response
2	Correctly identifies necessary integrals
1	Work towards solution
Marker's Comment	Many students mixed up composite areas with the method to find the area between curves. This question should have been broken into 3 separate areas. Many students failed to realise the area from x=3 to x=5 is not negative. Tip! Some areas can be found without using integration by considering the area of a simple shape; in this case a triangle.

Question 28   (4 marks)
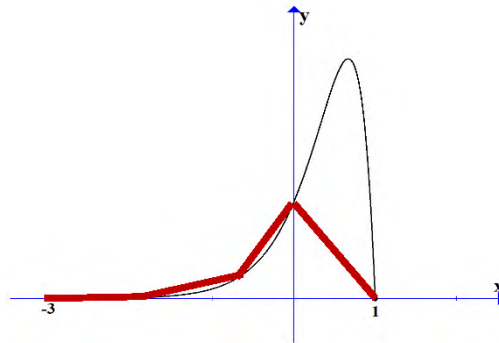
Consider the function defined by $f(x) = e^{3x}(2 - 2x)$ where $-3 \leq x \leq 1$.

a) Complete the table of values below. Give values correct to 3 decimal places.   **1**

x	-3	-2	-1	0	1
$f(x)$	0.001	0.015	0.199	2.000	1.000

b) Using the trapezoidal rule with five function values, approximate the area under the curve   **2**
    $y = f(x)$ for $-3 \leq x \leq 1$

$A \approx \dfrac{\sim 1 - -3}{2 \times 4}\{0.001 + 0 + 2 \times [0.015 + 0.199 + 2]\}$

$\approx \dfrac{1}{2}\{4.429\}$

≈ 2.2145

Marks	Guideline
2	Correct solution
1	For progress towards solution
Marker's Comment	Many students confused the value for *n* in the formula with the number of function values rather than the number of sub-intervals. Students should be aware that if their application of the trapezoidal rule provides a negative answer then they have made an error. Don't take the absolute value of your answer (that's when integrating to find the area below the x-axis). TIP! Use the reference sheet.

c) From the diagram, decide whether this approximation is an over-estimate or an under-estimate   **1**
    of the true value of the area under the curve. Justify your answer.

Underestimate. By connecting the function values we can deduce that the peak of curve is cut off due to the curve being concave down between 0 and 1.

Note: the actual area is ≈ 4.46

Marks	Guideline
1	Correct answer
Marker's Comment	It was not enough to deduce that it was an underestimate. Students had to *justify* why. While reasons may have been clear to the students, many had trouble in expressing the correct concepts. TIP! Draw the subintervals over the graph.

Question 29   (5 marks)

A railway line is being built between Albury and Ballarat. As of the 1ˢᵗ of January 2023, 80km has been built. Each month 15km of new track is built.

a) Find the total length of the track 1 month, 2 months and 3 months after the 1ˢᵗ of January 2023. Hence, show that this is an arithmetic sequence.   **2**

95, 110, 125
$T_2 - T_1 = 15$
$T_3 - T_2 = 15$ $\therefore$ *arithmetic sequence*

Marks	Guideline
2	Correct solution
1	For calculating first 3 terms OR for demonstrating the property of an arithmetic sequence
Marker's Comment	Most students were able to add 15 to the previous months distance to get the required terms. Better responses included the subtraction of terms to prove it was an arithmetic sequence.

b) Find an expression for the total length of the track n months after the 1ˢᵗ of January 2023   **1**

$T_n = 95 + (n-1)15$
$\quad = 80 + 15n$

Marks	Guideline
1	Correct answer
Marker's Comment	There was some confusion over whether this was a sum or a term question and what was the first term.

c) The distance from Albury to Ballarat is 392km. In what month and year will the railway line be finished?   **2**

$392 = 80 + 15n$
$15n = 312$
$n = 20.8 months$
$20.8 - 12 = 8.8$ months after January in 2024
September 2024

Marks	Guideline
2	Correct solution
1	Calculates n OEM
Marker's Comment	It appeared as though some students ignored the answer from part b and just started again. Students who got 21.8 months struggled to convert this to Oct 2024 instead of September 2024.

Question 30 (3 marks)

The graphs of $f(x) = -x^2 + 5x$ and $g(x) = x^2 - 9x + 12$ are shown graphed below. Find the area of the shaded region between the 2 curves.
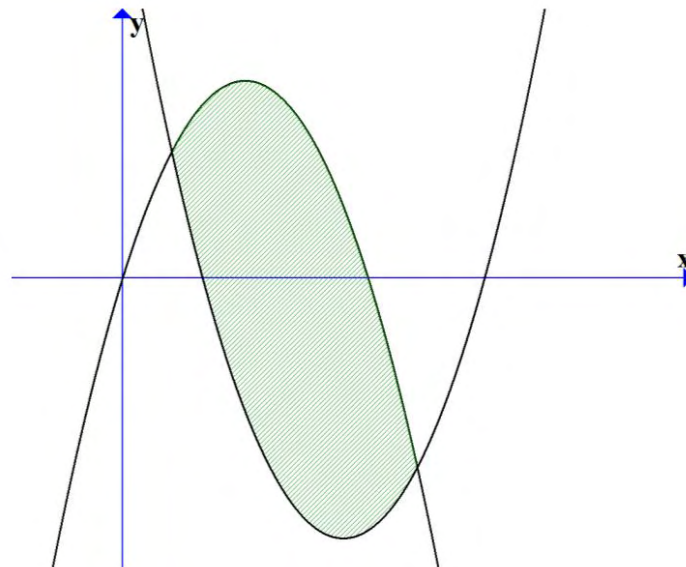
$$-x^2 + 5x - (x^2 - 9x + 12)$$
$$-2x^2 + 14x - 12$$
$$-x^2 + 5x = x^2 - 9x + 12$$
$$2x^2 - 14x + 12 = 0$$
$$2(x^2 - 7x + 6) = 0$$
$$2(x - 6)(x - 1) = 0$$
$$x = 6, 1$$

$$\int_1^6 -2x^2 + 14x - 12\, dx$$
$$= \left[\frac{-2x^3}{3} + \frac{14x^2}{2} - 12x \right]_1^6$$
$$= \frac{-2 \times 6^3}{3} + 7 \times 6^2 - 12 \times 6 - \left(\frac{-2}{3} + 8 - 12 \right)$$
$$= \frac{125}{3}\, units^2$$

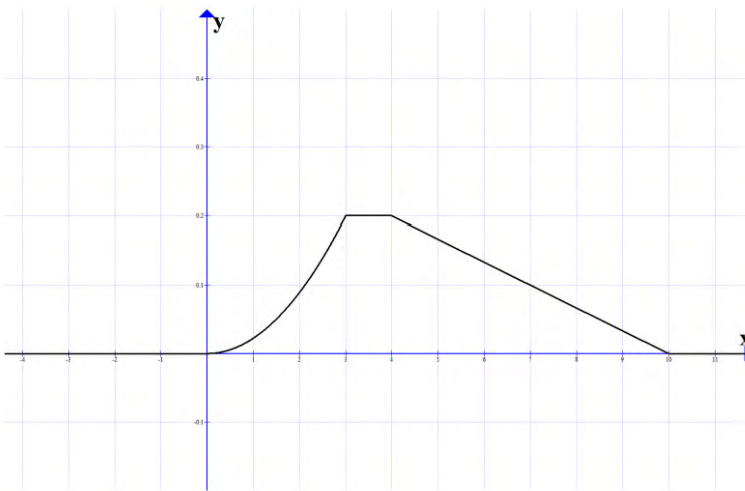| Marks	Guideline
3	Correct response
2	Correctly identifies necessary integrals
1	Work towards solution
Marker's Comment	- Most students were able to solve the equations simultaneously to find intersections at X=1 and 6. - Better responses knew to find the area between two curves you just subtracted the function and integrated the simplified form. - Students made errors when they forgot to include brackets when subtracting and hence integrated the wrong integral. i.e. $-x^2 + 5x - x^2 - 9x + 12$ instead of $(-x^2 + 5x) - (x^2 - 9x + 12)$ leading to the x term having the wrong sign and coefficient, and the constant having the wrong sign.

Question 31   (4 marks)

The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{x^2}{45} & 0 \le x \le 3 \\[2mm] \dfrac{1}{5} & 3 < x < 4 \\[2mm] \dfrac{1}{3} - \dfrac{x}{30} & 4 \le x \le 10 \\[2mm] 0 & \text{otherwise} \end{cases}$$

a) Sketch $f(x)$ for $0 \le x \le 10$

Marks	Guideline
2	All correct graphs drawn neatly and clearly
1	At least one part of the function drawn correctly
Marker's Comment	Students should use a ruler to draw straight liners. Each part of the piece-wise function should have been easy to draw as it was only early Yr11 work.

b) Find $P(X \le 8)$ **2**

Method 1:

$$P(x \le 8) = \int_0^3 \frac{x^2}{45}\,dx + \int_3^4 \frac{1}{5}\,dx + \int_4^8 \frac{1}{3} - \frac{x}{30}\,dx$$

$$= \left[\frac{x^3}{135}\right]_0^3 + \left[\frac{x}{5}\right]_3^4 + \left[\frac{x}{3} - \frac{x^2}{60}\right]_4^8$$

$$= \frac{3^3}{135} + \left[\frac{4}{5} - \frac{3}{5}\right] + \left[\frac{8}{3} - \frac{64}{60} - \left(\frac{4}{3} - \frac{16}{60}\right)\right]$$

$$= \frac{14}{15}\,units^2$$

Marks	Guideline
2	Correct answer with appropriate working
1	Work towards solution
Marker's Comment	Generally well done. Suggest students look at what they are finding the area of and use any shortcuts, i.e. area of rectangle/triangle. Be careful of calculation errors, such as $3^3 = 9$ instead of $3^3 = 27$.

Method 2:

$$P(x \le 8) = 1 - P(x > 8)$$

$$= 1 - \frac{1}{2} \times 2 \times \left(\frac{1}{3} - \frac{8}{30}\right)$$

$$= \frac{14}{15}\,units^2$$